Can you measure the photon rest mass with an ion interferometer?

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We propose using a table-top ion interferometer to search for deviations from Coulomb's inverse-square law. Calculations show that such a device would be sensitive to deviations smaller than 6×10^{-22} in the exponent of the inverse-square law, an improvement by five orders of magnitude over current experiments. It could measure a non-zero photon rest mass smaller than 6×10^{-50} grams, more than 100 times smaller than current laboratory experiments. We also discuss the theory behind the proposed measurement and compare our predictions to previous studies.

Matter-wave interferometry is a very useful tool to test fundamental physical theories. One such theory, Coulomb's inverse square law, has been under scrutiny since 1769 [1, 2, 3]. In the analysis of these experiments the potential of a point charge was assumed fall to off as $r^{-(1+\delta)}$. By careful measurements, the most recent experiment suggests that δ must be smaller than 6×10^{-17} [4]. We show that a table-top ion interferometer could improve this limit by many orders of magnitude.

Additional motivation for the search for Coulomb's law violations spawned from the application of Proca's theory of massive spin-one particles to electromagnetism [3], resulting in a modified version of Maxwell's equations parameterized by a non-zero photon rest mass. These equations predict things such as light dispersion in vacuum, a longitudinal component of light polarization, and non-zero net charge distributions inside of conductors [5, 6]. In this framework the photon rest mass can be determined by measuring a non-zero electric field inside a conducting shell.

Because the rest mass of a photon is a common parameter by which a broader range of experiments can be related, and because the Proca equations are well established and derived from a more fundamental test theory [6], we will primarily use the Proca formalism in our discussion. All of the experimental parameters will be optimised for measurements of photon mass. But if a non-zero field is found, Proca theory is not the only possible way to explain it. As such, the photon rest mass should not be considered an absolute measure with which to compare vastly different experiments.

In the proposed experiment, a possible Coulomb's-law violating field inside of a conducting cylinder is measured using an ion interferometer. As shown in Fig. 1, ions travel through a long conducting tube nested inside of a second tube. The outer conductor is grounded to provide an unchanging reference, and a time-varying voltage is applied to the inner conductor. A beam of cold atoms, shown on the left of the figure, passes through small holes in the conductors. The atoms are ionized, possibly with a laser beam shown as an up-pointing arrow in the figure, and pass through three gratings to form a Mach-Zehnder interferometer. These could be physical [7] or light gratings [8]. If an electric field is present in the

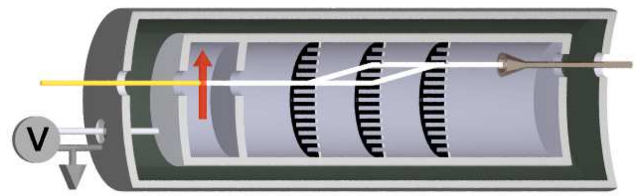

Figure 1: (Color online) A cut-away schematic of the proposed experiment. Diagram is not to scale, and some dimensions have been greatly exaggerated for visibility.

inner conductor, the two arms of the interferometer will pass through different potentials, resulting in a relative phase shift. This phase can be measured by detecting the fraction of ions blocked or passed by the final grating.

To calculate the limit on the photon rest mass that could be achieved in such an apparatus, we start with a modified version of Laplace's equation:

$$\nabla^2 \phi - \mu_\gamma^2 \phi = 0. \tag{1}$$

This equation is derived from the modified versions of Maxwell's equations generated by the Proca action for massive photons. In this equation ϕ is the scalar electrostatic potential, and the small constant μ_γ is related to the photon rest mass m_γ by the relation $m_\gamma = \mu_\gamma \hbar / c$ where $\hbar$ is Planck's constant divided by 2π, and c is the speed of light in vacuum.

In the limit as $\mu_\gamma \to 0$, Eq. 1 becomes Laplace's equation. For the simple case of a spherically symmetric system, Laplace's equation has the familiar solutions $\phi(r) = A/r$ and $\phi(r) = B$, where A and B are constants. The A/r solution is the familiar point-charge potential. The constant B solution allows us to arbitrarily define a point to be at zero potential without changing the fields described by the potential. If $\mu_\gamma \neq 0$, the solutions for a spherically symmetric system are a Yukawa potential $\phi(r) = (A/r)\exp(-\mu_\gamma r)$, and an exponentially growing solution $(B/r)\exp(\mu_\gamma r)$. The Yukawa potential solution lets us interpret $1/\mu_\gamma$ as an effective range of the Coulomb force. Without a constant solution, absolute potential

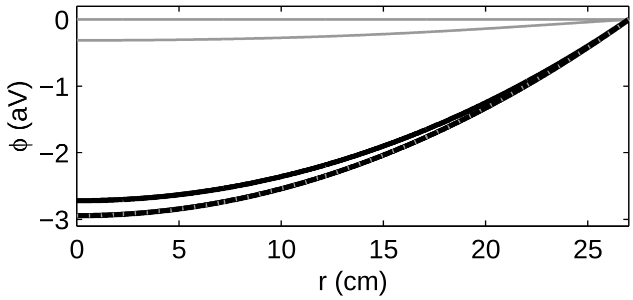

Figure 2: Numerical calculations of potentials in a 2.6 m long, 27 cm radius tube held at 200 kV. The calculation assumes 20 cm long end caps on the tube to reduce fringing fields, and m_γ is assumed to be 1×10^{-50} grams. An off-center round hole in each end cap would allow the ions to enter the tube and would accomodate the detector. To simplify the calculation, the holes were replaced with ring-shaped apertures with an inner and outer radius of 24.75 and 25.25 cm. Potentials are plotted in attovolts vs. the radial distance from the tube axis. The black lines show the calculated deviation from the classical potential at axial distances of zero (lower line) and one meter (upper line) from the middle of the tube, and the dotted grey line is the deviation expected for an infinite tube. The solid grey lines show the calculated classical fringing-field potentials at zero and one meter from the middle of the tube, both multiplied by 10^{35} to make them visible on this scale.

has physical significance and we are no longer free to arbitrarily choose where ϕ equals zero.

Due to the elongated geometry of the proposed experiment, we will approximate the finite inner conductor with an infinitely long tube. Numerical and analytical studies have verified that this is a good approximation for reasonably long tubes (see Fig. 2). For a system with no angular or longitudinal dependence, solutions to Eq. 1 have the form

$$\phi(r) = CI_0(\mu_\gamma r) + DK_0(\mu_\gamma r), \qquad (2)$$

where C and D are constants and $I_0(x)$ and $K_0(x)$ are the zeroth-order modified Bessel functions. Because $K_0(\mu_\gamma r)$ diverges as $r \to 0$, we know that $D = 0$ inside the inner tube.

To find C we apply the condition that $\phi(r)$ must equal the applied voltage when $r = R$ (the radius of the tube). We will write this voltage as $V + V_g$, where V is the voltage applied to the inner tube relative to the outer tube, and V_g is the unknown voltage of the outer, grounded tube. Then, because μ_γ is known to be small, we can approximate the potential inside the inner tube with a lowest-order Taylor series:

$$\phi(r) \approx (V + V_g)\left[1 + \frac{\mu_\gamma^2}{4}(r^2 - R^2)\right]. \qquad (3)$$

Rather than absolute potential, the interferometer will measure the potential difference between the two arms. Each of the arms in Fig. 1 consists of one horizontal and one diagonal segment. Both diagonal segments pass through identical potentials which induce equal phase shifts on the upper and lower arms. As such, the diagonal segments can be neglected. The horizontal segments, however, are at two different radii and travel through different potentials. Assuming that the lower and upper horizontal segments are a distance r_0 and $r_0 + s$ from the center of the tube, the potential difference between them is

$$\Delta\phi = \phi(r_0 + s) - \phi(r_0) \approx \frac{\mu_\gamma^2}{4}(V + V_g)\left(s^2 + 2r_0 s\right). \qquad (4)$$

If τ is the time that it takes the ions to travel the length of the horizontal segments, and e is the ion charge, the interferometer phase Φ is given by

$$\Phi \approx \frac{e\mu_\gamma^2}{4}(V + V_g)\left(s^2 + 2r_0 s\right)\frac{\tau}{\hbar}. \qquad (5)$$

Because we don't know V_g, and because many factors can offset the absolute phase of an interferometer, we would apply a time-varying potential V and look for a correlated change in the interferometer phase. For example, imagine that the applied potential V is periodically reversed. Because the Earth has a very large capacitance, V_g will remain constant, and the difference in phase when V is reversed will be

$$\Delta\Phi \approx \frac{eV\mu_\gamma^2}{2}\left(s^2 + 2r_0 s\right)\frac{\tau}{\hbar}. \qquad (6)$$

Solving Eq. 6 for μ_γ we can determine the rest mass of the photon from the measured interferometer phase shift:

$$m_\gamma \approx \frac{\hbar}{c}\left[\frac{2\hbar\Delta\Phi}{eV\left(s^2 + 2r_0 s\right)\tau}\right]^{1/2}. \qquad (7)$$

To estimate the smallest detectable m_γ, we insert reasonable experimental parameters into Eq. 7. One important parameter is the velocity of the ions v. If v is small, the ions will diffract at larger angles. But if v is too small, stray electric fields can have a significant effect on ion velocities and trajectories. These fields will be extremely small inside the tube at the locations of the gratings (see Fig. 2), but they could be much larger in the region where the ions are generated. We can write the velocity as $v = (2eV_s/m)^{1/2}$ where m is the mass of the ions and V_s is the voltage which would just bring the ions to a stop. Then we can set V_s to be several times the level of the expected stray fields to be sure that the trajectory of the ions is not greatly perturbed by them.

Two other important parameters are the the maximum excursion of the ions from the center of the tube, $a = r_0 + s$, and the distance between gratings, L. A larger tube radius accommodates a larger separation s and offset r_0. A larger grating separation L means that the ions will interact with the field longer ($\tau = L/v$) and results in a greater separation of the two arms of the interferometer ($s \approx Lh/mvd$, where where h is Planck's constant and d is the grating period). With these parameters in mind, we can rewrite Eq. 7 as

$$m_\gamma \approx \frac{\hbar}{cL}\left[\frac{\Delta\Phi d V_s}{\pi V a\,(1-Q)}\right]^{1/2} \tag{8}$$

where $Q = \pi L\hbar/(2emV_s a^2 d^2)^{1/2}$. This equation shows very weak dependence on ion mass and charge — although higher charge and lower mass results in more precision for a given ion velocity, this is offset by the greater velocity needed to overcome deflections by stray fields. For the parameters selected below, Q is small for all possible ion masses, and the ultimate precision of the experiment will not change much with the mass or charge of the ion.

There are practical limits on L and a for a table-top apparatus. We chose L to be one meter, and limited a to be a conservative 25 cm. For our numerical calculations (Fig. 2) we assumed a total length for the inner tube plus end-caps of 3 m, and a tube radius of 27 cm. This gives sufficient space to limit fringing fields, to be sure that the infinite-tube calculation is a good approximation in the region of the interferometer, and keeps the ions in the interferometer from traveling close to the tube surface. Only a, and not the outer radius of the tube affect the precision predicted in Eq. 8, so a tube with a larger radius could be used to further limit ion-surface interactions without reducing the predicted precision.

We assumed a grating period of 100 nm, equal to the gratings in [7]. The applied voltage V can be generated directly, or the two tubes can be used as a capacitor in a tank circuit. We selected a value of 200 kV because it is within the range of what is possible with off-the-shelf power supplies and vacuum feed-throughs. Based on work done with atom interferometers [9], it should be possible to directly detect phase-shifts as small as 10^{-4} radians. We set the final parameter, V_s, to 0.5 mV assuming that voltages due to stray fields can be controlled at a level much smaller than this.

With these parameters we predict a sensitivity to photon rest mass of 6×10^{-50} grams. Precision could be increased by using a larger apparatus or by rapidly varying the applied voltage and using a phase-sensitive lock-in technique. The ultimate limit for such a device depends on the ion flux and the stability of systematic effects. To the extent that systematic shifts are stable over the time that it takes to reverse the voltage they will cancel out. The largest drifts are expected to be due to inertial-force shifts. Patch-charge effects and effects due to the charging of the gratings by the ion beam should be stable in steady state, and not significantly affected by the voltage reversal. Numerical and analytical calculations show that fringing fields from holes in the conductor would be negligible at this level (see Fig. 2).

Given the assumed parameters, depending on the type of ion used the separation s would range from 13 mm (for $^1\mathrm{H}^+$) to 1.1 mm (for $^{133}\mathrm{Cs}^+$), and the ion beam would enter the apparatus at a radius r_0 ranging from 23.7 cm ($^1\mathrm{H}^+$) to 24.9 cm ($^{133}\mathrm{Cs}^+$). These parameters yield an s for electrons which is greater than the radius of the tube, so for these experimental parameters electrons are not an ideal choice. For a horizontal apparatus in gravity, assuming that the ions undergo a parabolic trajectory with the peak at the location of the center grating, the ions will fall a vertical distance ranging from 51 μm ($^1\mathrm{H}^+$) to 6.8 mm ($^{133}\mathrm{Cs}^+$), giving them a vertical velocity which is from 1.0×10^{-4} ($^1\mathrm{H}^+$) to 1.4×10^{-2} ($^{133}\mathrm{Cs}^+$) times their longitudinal velocity. As such, gravity should not be a major limitation, especially if a lighter ion is used.

Obtaining a precision of 6×10^{-50} grams requires the use of slow ions. These ions would be generated from a slow neutral atom beam. The velocity, determined by V_s and the mass of the ion, ranges from 311 m/s ($^1\mathrm{H}^+$) to 27 m/s ($^{133}\mathrm{Cs}^+$). The latter is a reasonable velocity for a beam of atoms from an LVIS source [10]. Higher velocities are easily obtained by accelerating the ions with a small potential. As such, any atom that can be laser cooled could be used to obtain the above stated sensitivity. However, lighter ions have the advantage of faster transit, which would make it possible to modulate the voltage applied to the tube at a higher frequency, reducing the effective bandwidth of systematic drifts.

If one employs an ion such as Ca^+ which has a resonant transition which is easily generated with current laser technology, an off-resonance optical grating could be used in place of physical gratings. For the example of calcium the wavelength of the transition is near 400 nm, resulting in a grating period of 200 nm — just twice the one assumed above. This would reduce the predicted sensitivity of the device by only a factor of $\sqrt{2}$ while eliminating potential problems due to surface charges on the gratings.

The predicted precision is considerably better than that reported previously for laboratory experiments. Furthermore, a potential pitfall is avoided. In the most recent experiments [4, 11] a voltage was applied across two conducting shells, and the voltage between one of the shells and a third shell was measured. Any non-zero electric field would tend to draw a charge onto the third shell to cancel the field. If a tiny amount of charge (about 0.001 times the charge of an electron) passed through the probe electronics to the third shell, it would cancel the field due to a photon mass larger than the reported precision. In our scheme the only influence the ions have on the system under test is the induction of an image charge in the conductor. Because the image charge is independent of the applied voltage, this effect should not result in a measured phase shift.

The work described in [11] sets an upper bound on m_γ of 2×10^{-47} grams. In a more recent variant on this experiment, a bound of 8×10^{-48} grams is given [4], although it is mentioned only in passing in a paper which focused on photon mass experiments for instructional lab courses. This is a factor of 2.5 between two experiments separated by over a decade, followed by over a decade without any progress. Our calculations show that ion interferometry could improve this latest measurement by two orders of magnitude.

Assuming that the Proca formalism is correct, other effects can be used to search for finite photon rest mass. Studies have been done using astronomical distances to magnify these tiny effects. One example is the measurement of the dispersion of starlight [12]. Another example is a pair of torsional pendulum measurements [13, 14]. These torsional pendulum experiments are sometimes erroneously placed in the same category as other laboratory experiments. But to obtain the photon rest mass from their laboratory measurement, an estimate of the cosmic vector potential must be used, making these experiments more like the model-dependent astronomical studies than the previously discussed laboratory experiments.

Our predicted precision exceeds that of most astronomical studies. But a few, including analysis of dispersion of hydromagnetic waves in the Crab Nebula [15], plasma stability in the Coma Cluster [16], dissipation of large-scale magnetic fields in the Galaxy [17], and stability of the Magellanic Clouds [18], stand out by orders of magnitude. Nevertheless, lab measurements are interesting even if they do not exceed the stated results of astronomical studies because they do not involve significant assumptions and modeling. And if a variation from Coulomb's law exists for reasons other than those assumed in the Proca formalism, the link between the various astronomical and laboratory studies is broken, and each set of studies measuring different effects must be considered separately.

In addition to limits on the photon rest mass, following the tradition of Cavendish [2] it is also common to assume that the potential would fall off as $r^{-(1+\delta)}$ and to quote a limit on the size of δ. Because we don't know that the Proca treatment is correct, this additional figure of merit is valuable. Unfortunately the $r^{-(1+\delta)}$ point-charge potential does not come from an underlying theory. If such a theory existed, $r^{-(1+\delta)}$ would enter naturally as a solution to a modified version of Laplace's equation. At least one more solution, one which is finite at $r = 0$, must exist. Knowing just one of the solutions is not sufficient to determine charge distributions.

It appears that previous experiments calculated δ by integrating the point-charge potential over the classical charge distribution. But the unmodelled deviation of the true charge distribution from the classical distribution could have a big impact on the implied magnitude of δ. Furthermore, if the unknown equation is nonlinear, the potential cannot be related to an integral of point charges. It is also somewhat disturbing that in this formalism the size of δ affects the units of the permitivity ϵ_0.

Ignoring the above-stated concerns, we integrated the point-charge potential over the classical charge distribution for two infinite concentric tubes with radii of 27 and 30 cm. From this we predict that a limit on δ of 6 to 8×10^{-22} would be possible (depending on the ion used). This would represent an improvement of five orders of magnitude over the value of 6×10^{-17} reported in [4].

In conclusion, we have discussed the potential of ion interferometry in searches for violations of Coulomb's law. Calculations using reasonable parameters suggest that a table-top device should be able to measure photon rest mass at the level of 6×10^{-50} grams, and measure deviation in the exponent of Coulomb's inverse square law at the level of 6×10^{-22}, both representing an improvement of several orders of magnitude over current laboratory measurements. In addition, the apparatus would be immune to effects related to the modification of the field by the instrument used to measure it.

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